

Magnon mode truncation in a rung-dimerized asymmetric spin ladder

P. N. Bibikov

V. A. Fock Institute of Physics Sankt-Petersburg State University

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Abstract

An effective model is suggested for an asymmetric spin ladder with dimerized rungs. Magnon mode truncation originated from magnon decay (recently observed in the 1D compound IPA – CuCl₃) is naturally described within this model. Using Bethe Ansatz we studied a one-magnon sector and obtained relations between interaction constants of the model and experimentally observable quantities such as the gap and truncation energies, spin velocity and truncation wave vector. It is also shown that the structure factor turns to zero at the truncation point.

1 Introduction

A spin ladder with strong antiferromagnetic rung coupling gives an ideal example of a gapped spin-dimerized system [1]. Really the majority of spins in its ground state are coupled in rung-singlets (rung-dimers) so that the relative coupling energy estimates a value of the gap. By this reason all low-temperature effects depend on dynamical properties of states with a few number of excited rungs. Theoretical study of excitations in spin ladders with strong antiferromagnetic rung coupling was developed in a number of papers [2]-[5]. It was pointed out that the lowest excitations form a coherent magnon

branch. When the gap energy is smaller than the energy width of the magnon zone the latter may intersect the two-magnon scattering continuum. For a *symmetric* spin ladder (with equal couplings along both legs as well as along both diagonals) these two sectors do not hybridize so a one-magnon state is always stable. The situation is quite different for an *asymmetric* spin ladder with non equal couplings along legs or along diagonals. As it was pointed in [6]-[9] the coupling asymmetry entails hybridization between the "bare" (related to a symmetric case) one- and two-magnon sectors. If the system has a wide magnon band intersecting with the two-magnon scattering continuum this hybridization results to magnon instability and truncation of the magnon mode at some value k_{trunc} of wave vector. Experimentally a magnon mode truncation was observed in neutron scattering from 1D compound IPA – CuCl_3 ($((\text{CH}_3)_2\text{CHNH}_3\text{CuCl}_3)$ [6]. The latter is considered as an asymmetric spin ladder with strong ferromagnetic rungs and is effectively equivalent to a 1D Haldane antiferromagnet.

In this paper starting from an asymmetric rung-dimerized spin ladder we present an effective model which produce an explicit realization of magnon mode truncation related to magnon decay. Within our model we study the one-magnon excitations and obtain explicit relations between the coupling constants and experimentally observable quantities.

2 Hamiltonian for an asymmetric spin ladder

In the present paper we shall study an asymmetric spin ladder with the following Hamiltonian [4],[9],

$$\hat{H} = \sum_n H_{n,n+1}, \quad (1)$$

where $H_{n,n+1} = H_{n,n+1}^{rung} + H_{n,n+1}^{leg} + H_{n,n+1}^{diag} + H_{n,n+1}^{cyc} + H_{n,n+1}^{norm}$ and

$$\begin{aligned} H_{n,n+1}^{rung} &= \frac{J_{\perp}}{2} (\mathbf{S}_{1,n} \cdot \mathbf{S}_{2,n} + \mathbf{S}_{1,n+1} \cdot \mathbf{S}_{2,n+1}), \\ H_{n,n+1}^{leg} &= J_{\parallel} (\mathbf{S}_{1,n} \cdot \mathbf{S}_{1,n+1} + \mathbf{S}_{2,n} \cdot \mathbf{S}_{2,n+1}), \\ H_{n,n+1}^{diag} &= J_d \mathbf{S}_{1,n} \cdot \mathbf{S}_{2,n+1}, \\ H_{n,n+1}^{cyc} &= J_c ((\mathbf{S}_{1,n} \cdot \mathbf{S}_{1,n+1})(\mathbf{S}_{2,n} \cdot \mathbf{S}_{2,n+1}) + (\mathbf{S}_{1,n} \cdot \mathbf{S}_{2,n}) \\ &\quad \times (\mathbf{S}_{1,n+1} \cdot \mathbf{S}_{2,n+1}) - (\mathbf{S}_{1,n} \cdot \mathbf{S}_{2,n+1})(\mathbf{S}_{2,n} \cdot \mathbf{S}_{1,n+1})). \end{aligned} \quad (2)$$

Here $\mathbf{S}_{j,n}$ ($j = 1, 2$) are the $S = 1/2$ spin operators related to n -th rung. The auxiliary term $H^{norm} = J_{norm} I$ (I is an identity matrix) is added for the zero normalization of the ground state energy.

The following condition,

$$J_d + J_c = 2J_{\parallel}, \quad (3)$$

suggested in [4], guarantees that the vector, $|0\rangle_n \otimes |0\rangle_{n+1}$ (where $|0\rangle_n$ is the n -th rung-singlet or equivalently rung-dimer) is an eigenstate for $H_{n,n+1}$, so the vector

$$|0\rangle = \prod_n |0\rangle_n, \quad (4)$$

is an eigenstate for \hat{H} . An additional system of inequalities,

$$\begin{aligned} J_{\perp} &> 2J_{\parallel}, \quad J_{\perp} > \frac{5}{2}J_c, \quad J_{\perp} + J_{\parallel} > \frac{3}{4}J_c, \\ 3J_{\perp} - 2J_{\parallel} - J_c &> \sqrt{J_{\perp}^2 - 4J_{\perp}J_{\parallel} + 20J_{\parallel}^2 - 16J_{\parallel}J_c + 4J_c^2}, \end{aligned} \quad (5)$$

together with a condition $J_{norm} = 3/4J_{\perp} - 9/16J_c$, guarantee that the vector (4) is the (zero energy) ground state for \hat{H} . The full system of the "ground state tuning" conditions (3),(5) belongs to the mathematical basis of our model.

For $J_d = 0$ the Hamiltonian \hat{H} commutes with the operator, $\hat{Q} = \frac{1}{2} \sum_n (\mathbf{S}_{1,n} + \mathbf{S}_{2,n})^2$, considered as a number of "bare" magnons [5],[9]. Therefore the Hilbert space splits on an infinite sum: $\mathcal{H} = \sum_{m=0}^{\infty} \mathcal{H}^m$, where $\hat{Q}|_{\mathcal{H}^m} = m$. The subspace \mathcal{H}^0 is one-dimensional and generated by the ground state (4).

3 Spectral problem for the reduced Hamiltonian related to the effective model

Despite the ground state (4) for the Hamiltonian (1)-(3), (5) is known it is not clear how to obtain its excitations. In the symmetric case [4],[5] the one-magnon state corresponds to \mathcal{H}^1 but even for a small asymmetry it already lies in $\sum_{n=0}^{\infty} \mathcal{H}^{2n+1}$ [9]. By this reason the related spectral problem seems to be unsolvable. However for a strong rung coupling the states with rather big number of "bare" magnons have a large energy and therefore may be effectively reduced. In the first order with respect to the dimerization energy the reduced Hilbert space $\mathcal{H}^{red} = \mathcal{H}^0 \oplus \mathcal{H}^1 \oplus \mathcal{H}^2$ contains additionally to the ground state (4), only the "bare" one- and two-magnon sectors. The corresponding effective Hamiltonian \hat{H}^{eff} is defined as the restriction of \hat{H} on \mathcal{H}^{red} or,

$$\hat{H}^{eff} = P^{(0,1,2)} \hat{H} P^{(0,1,2)}, \quad (6)$$

where $P^{(0,1,2)}$ is the projector on \mathcal{H}^{red} .

A general $S = 1$ excited state for \hat{H}^{eff} related to a wave vector k and the energy $E(k)$ is superposition of "bare" one- and two-magnon components,

$$|k\rangle^\alpha = \frac{1}{Z(k)\sqrt{N}} \sum_m [a(k)e^{ikm} \dots |1\rangle_m^\alpha \dots + \varepsilon_{\alpha\beta\gamma} \sum_{n>m} e^{ik(m+n)/2} b(k, n-m) \dots |1\rangle_m^\beta \dots |1\rangle_n^\gamma \dots], \quad (7)$$

where $|1\rangle_n^\alpha = (\mathbf{S}_{1,n}^\alpha - \mathbf{S}_{2,n}^\alpha)|0\rangle_n$ and " \dots " means an infinite product of dimers related to the remaining rungs. The normalization factor $Z(k)$ is defined as,

$$Z^2(k) = |a(k)|^2 + 2 \sum_{n=1}^{\infty} |b(k, n)|^2. \quad (8)$$

The system of Shrödinger equations on the amplitudes $a(k)$ and $b(k, n)$ directly follows from the local action of the operator $H_{n,n+1}$,

$$\begin{aligned} H_{n,n+1}|0\rangle_n|1\rangle_{n+1}^\alpha &= \left(\frac{1}{2}J_\perp - \frac{3}{4}J_c\right)|0\rangle_n|1\rangle_{n+1}^\alpha + \frac{J_c}{2}|1\rangle_n^\alpha|0\rangle_{n+1} - \frac{iJ_d}{2}\varepsilon_{\alpha\beta\gamma}|1\rangle_n^\beta|1\rangle_{n+1}^\gamma, \\ H_{n,n+1}|1\rangle_n^\alpha|0\rangle_{n+1} &= \left(\frac{1}{2}J_\perp - \frac{3}{4}J_c\right)|1\rangle_n^\alpha|0\rangle_{n+1} + \frac{J_c}{2}|0\rangle_n|1\rangle_{n+1}^\alpha, \\ H_{n,n+1}\varepsilon_{\alpha\beta\gamma}|1\rangle_n^\beta|1\rangle_{n+1}^\gamma &= (J_\perp - J_\parallel - J_c/4)\varepsilon_{\alpha\beta\gamma}|1\rangle_n^\beta|1\rangle_{n+1}^\gamma + iJ_d|0\rangle_n|1\rangle_{n+1}^\alpha. \end{aligned} \quad (9)$$

From (7) and (9) one can obtain an infinite set of recurrent equations,

$$(2J_\perp - 3J_c)b(k, n) + J_c \cos \frac{k}{2} [b(k, n-1) + b(k, n+1)] = E(k)b(k, n), \quad n > 1, \quad (10)$$

related to non- neighbor excited rungs and two additional equations related to neighbor rungs,

$$\begin{aligned} (J_\perp - \frac{3}{2}J_c + J_c \cos k)a(k) + iJ_d \cos \frac{k}{2}b(k, 1) &= E(k)a(k), \\ (2J_\perp - \frac{9}{4}J_c - \frac{J_d}{2})b(k, 1) + J_c \cos \frac{k}{2}b(k, 2) - \frac{iJ_d}{2} \cos \frac{k}{2}a(k) &= E(k)b(k, 1). \end{aligned} \quad (11)$$

For a coherent excitation originated from the hybridization of the one-magnon and bound two-magnon states there must be,

$$\lim_{n \rightarrow \infty} b(k, n) = 0. \quad (12)$$

With regard to this condition the Eq. (10) has the following general solution,

$$b(k, n) = B(k)z^n(k), \quad (13)$$

where,

$$|z(k)| < 1. \quad (14)$$

and

$$E(k) = 2J_{\perp} - 3J_c + J_c \left(z(k) + \frac{1}{z(k)} \right) \cos \frac{k}{2}. \quad (15)$$

From (14) and (15) follows that,

$$\text{Im } z(k) = 0. \quad (16)$$

Substituting (13) and (15) into (11) we obtain a pair of equations on $a(k)$ and $B(k)$ represented in the following matrix form,

$$M(k) \begin{pmatrix} a(k) \\ B(k) \end{pmatrix} = 0, \quad (17)$$

where

$$M(k) = \begin{pmatrix} \frac{3}{2}J_c + J_c \cos k - J_c \left(z(k) + \frac{1}{z(k)} \right) \cos \frac{k}{2} - J_{\perp} & iz(k)J_d \cos \frac{k}{2} \\ -\frac{iJ_d}{2} \cos \frac{k}{2} & \left(\frac{3}{4}J_c - \frac{J_d}{2} \right) z(k) - J_c \cos \frac{k}{2} \end{pmatrix}. \quad (18)$$

The Eq. (17) is solvable only for $\det M(k) = 0$, or,

$$\begin{aligned} & \left[z^2(k)J_c \cos \frac{k}{2} + \left(J_{\perp} - \frac{3}{2}J_c - J_c \cos k \right) z(k) + J_c \cos \frac{k}{2} \right] \\ & \times \left[\left(\frac{3}{2}J_c - J_d \right) z(k) - 2J_c \cos \frac{k}{2} \right] + z^2(k)J_d^2 \cos^2 \frac{k}{2} = 0. \end{aligned} \quad (19)$$

The Eq. (19) added by the conditions (14) and (16) completely defines the coherent spectrum for \hat{H}^{eff} . The truncation originates from a failure of any of the conditions (14) or (16). For the first possibility the truncation wave vector k_{trunc} coincides with the, critical wave vector k_c defined as,

$$|z(k_c)| = 1. \quad (20)$$

For the second one it coincides with the branching wave vector k_b related to passing of solutions of the Eq. (19) into the complex plane.

In order to clear the nature of the truncation point for an arbitrary set of coupling parameters (however limited by (3) and (5)) let us first examine the case when the condition (20) is satisfied just at the branching point. In other words we are interesting in $k_c = k_b = k_{bc}$ when the Eq. (19) has a twice-degenerate solution $z(k_{bc})$ so that the same one has the equation obtained from (19) by differentiating of its left side with respect to

$z(k)$. Using an auxiliary variable $f = z(k_{bc}) \cos \frac{k_{bc}}{2}$ and taking in mind that according to (16) and (20) $z^2(k_{bc}) = 1$, we represent (at $k = k_{bc}$) the Eq. (19) and its "derivative" equation as the following system,

$$\begin{aligned} 4J_c^2 f^3 + (J_d^2 + 2J_d J_c - 7J_c^2) f^2 + J_c(4J_c - 2J_d - 2J_\perp) f + \left(\frac{3}{2}J_c - J_d\right)\left(J_\perp - \frac{J_c}{2}\right) &= 0, \\ 2J_c^2 f^3 + (J_d^2 + 2J_d J_c - 5J_c^2) f^2 + J_c\left(\frac{7}{2}J_c - 2J_d - J_\perp\right) f + \left(\frac{3}{2}J_c - J_d\right)\left(J_\perp - \frac{J_c}{2}\right) &= 0, \end{aligned} \quad (21)$$

which is solvable only for,

$$J_c J_d (2J_\perp - J_c) (3J_c - 2J_d) = 0. \quad (22)$$

(The left side of (22) was obtained from the resultant of the two polynomials in the left sides of (21)). The solution $J_c = 0$ of the Eq. (22) is not interesting because in this case the Eq. (19) is singular and solvable only for $k = \pi$. The solution $J_c = 2J_\perp$ is inconsistent with (5). The solution $3J_c = 2J_d$ is artificial because in this case $\cos \frac{k}{2}$ factorizes from the left side of (19), and therefore at $k = \pi$ the Eq. (19) is identically satisfied for all $z(\pi)$. The solution $J_d = 0$ relates to zero asymmetry when the corresponding truncation wave vector k_0 ,

$$\cos \frac{k_0}{2} = \frac{1}{2} \left(\sqrt{\frac{2J_\perp}{J_c}} - 1 \right), \quad (23)$$

may be easily obtained from the Eq. (19) which also gives,

$$z(k_0) = -1. \quad (24)$$

The formula (23) has a clear physical interpretation. Really as it follows from the results of the Refs. [4] and [5] (related to *symmetric* spin ladders) at $k = k_0$ the "bare" one-magnon branch with dispersion $E_{bare}^{magn}(k) = J_\perp - \frac{3}{2}J_c + J_c \cos k$ intersects the lower bound of the scattering two magnon continuum [5],

$$E_{bare}^{2magn,low}(k) = 2J_\perp - 3J_c - 2J_c \cos \frac{k}{2}. \quad (25)$$

The above result confirm the general statement suggested in [6]-[8] that even an extremely small asymmetry may change drastically a magnon mode. As it follows from (23) at $J_d \rightarrow 0$ the truncation occurs only for $9J_c > 2J_\perp$. Since $J_\perp > 0$ the parameter J_c also must be positive.

In order to find a nature of the truncation at $J_d \neq 0$ let us study an evolution of $z(k_b)$ for small J_d . If the condition (14) is satisfied for $k = k_b$ then the truncation originates

from branching and $k_{trunc} = k_b$. However in the opposite side for $|z(k_b)| > 1$ it will be $k_{trunc} = k_c$.

Taking for $J_d/J_c \ll 1$ and $k \approx k_0$ the following infinitesimal representation $z(k) = -1 + \epsilon(k)$, using the following notations $t(k) = \cos k/2$, $t_0 = \cos k_0/2$ and the formula,

$$\frac{J_\perp}{J_c} - \frac{3}{2} - \cos k = 2(t_0^2 + t_0 - t^2(k)), \quad (26)$$

which follows from (23) we obtain from (19) by omitting the term $\epsilon^3(k)$ the following equation,

$$\alpha(k)\epsilon^2(k) + \beta(k)\epsilon(k) + \gamma(k) = 0. \quad (27)$$

Here

$$\begin{aligned} \alpha(k) &= 1 + \frac{t(k)}{\Delta_1} + 2 \frac{(t(k) - t_0)(t(k) + t_0 + 1)}{t(k)} - \frac{J_d^2 t(k)}{2J_c^2 \Delta_1}, \\ \beta(k) &= 2 \frac{(t_0 - t(k))(t(k) + t_0 + 1)}{t(k)} \left(2 + \frac{t(k)}{\Delta_1}\right) + \frac{J_d^2 t(k)}{J_c^2 \Delta_1}, \\ \gamma(k) &= 2 \frac{(t(k) - t_0)(t(k) + t_0 + 1)}{t(k)} \left(1 + \frac{t(k)}{\Delta_1}\right) - \frac{J_d^2 t(k)}{2J_c^2 \Delta_1}, \end{aligned} \quad (28)$$

and $\Delta_1 = 3/4 - J_d/(2J_c)$.

The branching wave vector is characterized by the following condition,

$$D(k_b) = \beta^2(k_b) - 4\alpha(k_b)\gamma(k_b) = 0. \quad (29)$$

After its linearization with respect to small parameters $t(k) - t_0$ and J_d^2/J_c^2 this equation reduces at first to, $\gamma(k_b) = 0$ and then to,

$$\cos \frac{k_b}{2} \approx t_0 + \frac{J_d^2 t_0^2}{4J_c^2 (2t_0 + 1)(t_0 + \Delta_1)}. \quad (30)$$

According to (27) and (29), $\epsilon(k_b) = -\beta(k_b)/(2\alpha(k_b))$, or using (30) and (28),

$$\epsilon(k_b) \approx -\left(\frac{J_d t_0}{2J_c(t_0 + \Delta_1)}\right)^2. \quad (31)$$

Since $\epsilon(k_b) < 0$ the condition (14) fails for $z(k_b)$. Therefore for $J_d^2 \ll J_c^2$ there must be,

$$k_{trunc} = k_c. \quad (32)$$

Since for $J_d \neq 0$ the wave vector k_c evolve continuously from k_0 the Eqs. (24), (20) and (16) give,

$$z(k_c) = -1. \quad (33)$$

Despite the Eqs. (32) and (33) were proved for $J_d^2 \ll J_c^2$ they are right for all J_d . Really if for some region of J_d it will be $k_{trunc} = k_b$ then there must be a point where $k_c = k_b$. But as it was shown above k_0 is the only one point of such type.

The Eqs. (15) and (33) give the following representation for the magnon energy at the truncation point,

$$E_{trunc} = 2J_{\perp} - 3J_c - 2J_c \cos \frac{k_c}{2}. \quad (34)$$

The magnon branch approaches the bottom of the two-magnon continuum tangentially,

$$\left. \frac{\partial}{\partial k} E_{bare}^{2magn, low}(k) \right|_{k=k_c} = \left. \frac{\partial}{\partial k} E(k) \right|_{k=k_c} = J_c \sin \frac{k_c}{2}. \quad (35)$$

The Eq. (35) may be easily derived from (25) and (15) using an auxiliary relation,

$$\left. \frac{\partial}{\partial k} \left(z(k) + \frac{1}{z(k)} \right) \right|_{k=k_c} = 0, \quad (36)$$

which follows from (33). The same result was obtained in [7] by a different approach.

Let us notice that the singularity at $t_0 + \Delta_1 = 0$ in the formulas (30) and (31) originates from a resonance between the one-magnon and bound two-magnon states. Really for $t_0 = -\Delta_1$ the Eq. (19) has the thrice-degenerated solution related to both these states. This special case is not considered in the present paper.

4 Magnon dispersion near the gap

Let us turn to the opposite side of the spectrum related to $k = \pi$. As it follows from (19) $z(k)$ is an odd function and $z(\pi) = 0$. Therefore for $k \approx \pi$ we may put,

$$z(k) \approx z_1(\pi - k) + z_3(\pi - k)^3. \quad (37)$$

Then from (15) and (37) follows that for $z_1^3 - z_1/12 - z_3 > 0$ the dispersion at $k \approx \pi$ takes the form,

$$E(k) \approx E_{gap} \left(1 + \frac{v_{spin}^2}{2E_{gap}^2} (\pi - k)^2 \right), \quad (38)$$

where the gap energy E_{gap} and the spin velocity v_{spin} are given by

$$E_{gap} = 2J_{\perp} - 3J_c + \frac{J_c}{2z_1}, \quad \frac{v_{spin}}{E_{gap}} = \sqrt{\frac{J_c(z_1 - \frac{1}{12z_1} - \frac{z_3}{z_1^2})}{2J_{\perp} - 3J_c + \frac{J_c}{2z_1}}}. \quad (39)$$

Since both E_{gap} and v_{spin} may be obtained by an experiment [6] we shall express them explicitly from the coupling constants.

Substituting (37) into (19) we obtain the following system of equations on the coefficients z_1 and z_3 ,

$$\left[\left(J_{\perp} - \frac{J_c}{2}\right)z_1 + \frac{J_c}{2}\right]\left[\left(\frac{3}{2}J_c - J_d\right)z_1 - J_c\right] = 0, \quad (40)$$

$$\begin{aligned} &\left[\left(J_{\perp} - \frac{J_c}{2}\right)z_1 + \frac{J_c}{2}\right]\left[\left(\frac{3}{2}J_c - J_d\right)z_3 + \frac{J_c}{12}\right] \\ &+ \left[\frac{J_c}{2}\left(z_1^2 - z_1 - \frac{1}{12}\right) + z_3\left(J_{\perp} - \frac{J_c}{2}\right)\right]\left[\left(\frac{3}{2}J_c - J_d\right)z_1 - J_c\right] + \frac{J_d^2 z_1^2}{4} = 0. \end{aligned} \quad (41)$$

The Eq. (40) has two solutions,

$$z_1^{magn} = -\frac{J_c}{2J_{\perp} - J_c}, \quad z_1^{bound} = \frac{2J_c}{3J_c - 2J_d}, \quad (42)$$

related to magnon and bound two-magnon branches [4],[5]. According to the first equation in (42) $z_3^{magn}(J_{\perp} - J_c/2) = -J_c z_3^{magn}/(2z_1^{magn})$ so from (41) follows,

$$z_1^{magn} - \frac{1}{12z_1^{magn}} - \frac{z_3^{magn}}{(z_1^{magn})^2} = 1 - \frac{J_d^2}{J_c(4J_{\perp} + J_c - 2J_d)}, \quad (43)$$

and according to (39) one can obtain,

$$E_{gap} = J_{\perp} - \frac{5}{2}J_c, \quad \frac{v_{spin}}{E_{gap}} = \sqrt{\frac{2J_c}{2J_{\perp} - 5J_c} \left(1 - \frac{J_d^2}{J_c(4J_{\perp} + J_c - 2J_d)}\right)}. \quad (44)$$

As it follows from (44) the point $k = \pi$ corresponds to an energy minimum (the gap) only for $J_c(4J_{\perp} + J_c - 2J_d) > J_d^2$. (According to the comment after the Eq. (24) we suppose that $J_c > 0$.)

Using (33) and (34) we may represent the Eq. (19) in the point $k = k_c$ as follows,

$$\begin{aligned} &\left(\frac{(E_{trunc} - 2E_{gap}) \cos^2 \frac{k_c}{2}}{1 - \cos \frac{k_c}{2}} + E_{gap} - E_{trunc}\right) \\ &\times \left[\frac{(E_{trunc} - 2E_{gap}) \left(3 + 4 \cos \frac{k_c}{2}\right)}{4 \left(1 - \cos \frac{k_c}{2}\right)} - J_d\right] = J_d^2 \cos^2 \frac{k_c}{2}. \end{aligned} \quad (45)$$

where the parameters J_{\perp} and J_c are excluded by (34) and (44). The Eq. (45) may be used for obtaining the parameter J_d directly from an experimental data.

5 One-magnon dynamical structure factor near the threshold

We use the following representation for the dynamical structure factor (DSF),

$$S_{\alpha\beta}(\mathbf{q}, \omega) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{\mu} \langle 0 | \mathbf{S}^{\alpha}(\mathbf{q}) | \mu \rangle \langle \mu | \mathbf{S}^{\beta}(-\mathbf{q}) | 0 \rangle \delta(\omega - E_{\mu}). \quad (46)$$

Here $\mathbf{S}(\mathbf{q})$ is the spin Fourier transformation associated with the two dimensional vector $\mathbf{q} = (q, q_{rung})$ with leg and rung components. Since the latter has only two possible values 0 and π we may study them separately,

$$\mathbf{S}(q, 0) = \sum_n e^{-iqn} (\mathbf{S}_{1,n} + \mathbf{S}_{2,n}), \quad \mathbf{S}(q, \pi) = \sum_n e^{-iqn} (\mathbf{S}_{1,n} - \mathbf{S}_{2,n}). \quad (47)$$

According to the following pair of relations, $\mathbf{S}(q, 0)|0\rangle = 0$, $\mathbf{S}(-q, \pi)|0\rangle = \sum_n e^{iqn} \dots |1\rangle_n \dots$, we may reduce the matrix elements in (46)

$$\langle \mu | \mathbf{S}(q, 0) | 0 \rangle = 0, \quad \langle k | \mathbf{S}^{\beta}(-q, \pi) | 0 \rangle = \delta_{\alpha\beta} \delta_{kq} \frac{\sqrt{N} a(q)}{Z(q)}, \quad (48)$$

therefore, the DSF has purely diagonal form, $S_{\alpha\beta}(q, \pi, \omega) = \delta_{\alpha\beta} S(q, \pi, \omega)$, while the one-magnon contribution is purely coherent,

$$S_{magn}(q, \pi, \omega) = A_{magn}(q) \delta(\omega - E^{magn}(q)), \quad (49)$$

where

$$A_{magn}(q) = \left| \frac{a_{magn}(q)}{Z_{magn}(q)} \right|^2. \quad (50)$$

According to (8) and (13),

$$Z_{magn}(k) = \sqrt{|a_{magn}(k)|^2 + \frac{2|B_{magn}(k)|^2 z^2(k)}{1 - z^2(k)}}. \quad (51)$$

For $q \rightarrow k_c$, it will be $A_{magn}(q) \propto 1 - z^2(q)$, so as it follows from (33) $A_{magn}(k_c) = 0$. The same result was obtained in [7] by different approach.

Finally let us notice that a rather similar effect of hybridization between magnon and phonon modes was studied in [10]. However in the latter case a magnon mode does not truncate (because there is no decay) and therefore the corresponding structure factor does not turn to zero.

6 Summary and discussion

In this paper for a rung-dimerized asymmetric spin ladder we suggested an effective model which neglects all states with $n > 2$ bare magnons. Using Bethe Ansatz we studied the effect of magnon mode truncation resulting from magnon decay and clarified its mathematical nature (see the Eq. (32)). We obtained the four equations (see (34), (44) and (45)) coupling the interactions constants of our model (namely J_{\perp} , J_c and J_d) with the truncation wave vector, gap and truncation energies and spin velocity.

Of course the neglect of the states with $n > 2$ bare magnons is a rather rough approximation. Really an intersection between the one- and two-magnon scattering modes is possible only for a wide band system. In this case the bare $n > 2$ zones also lie not so far from the magnon mode and therefore give a rather essential contribution to it. However if we concern only on the gap and truncation points then our model produces a good approximation. Really near the gap the magnon energy is minimal and lies far below the bare $n > 2$ magnon modes. For example as it follows from (44) for $J_d \ll J_{\perp}, J_c$ even the $n = 2$ correction is small. From the other side since the $1 \rightarrow 2$ decay threshold lies on a *finite* distance below the $1 \rightarrow 3$ one the latter is not sufficient at the vicinity of the truncation point where the parameters $(E(k_c) - E(k))/E_{gap}$ and $(E(k_c) - E(k))/(E_{trunc} - E_{gap})$ are small. Therefore the infinitesimal analysis of the Sect. 3 (Eqs. (27)-(32)) gives the right picture of the truncation (the Eq. (33)).

There is only one known asymmetric rung-dimerized spin ladder compound namely the CuHpCl (see [11] and references therein). However the effect of truncation was not observed in this material. This fact is clear because the gap energy in CuHpCl (0.9 meV) is bigger than the magnon bandwidth (0.5 meV) so the magnon mode does not intersect with the scattering two magnon continuum.

Despite none wide-band asymmetric rung-dimerized spin ladder compound was found up to now we suppose that this may likely happen in not so remote future. Then the results of our paper probably will be useful for a theoretical study of such compound.

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